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The role of the entrance channel in the fusion of massive nuclei

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Abstract. The role of the entrance channel in the fusion-fission reactions leading to nearly the same superheavy compound nucleus is studied in the framework of dynamic model. The calculations are done for ^{48}Ca + ^{244}Pu and ^{74,76}Ge + ^{208}Pb reactions which could lead to formation of superheavy element $Z = 114$. It is shown that for these reactions there is an energy window for the values of the bombarding energy at which a capture probability is sufficiently large. Together with the restriction coming from the intrinsic barrier for fusion, it helps to find an optimal value of the bombarding energy for a given projectile–target combination.

PACS. 25.70.Gh Compoud nucleus – 25.70.Jj Fusion and fusion–fission reactions

1 Introduction

The cross section for the production of the superheavy elements depends on the choice of the projectile-target combination and the bombarding energy $E_{\rm c.m.}$. The optimal choice is determined by the requirements to have a larger fusion cross section and a larger survival probability of a compound nucleus relative to fission. For a given projectile-target combination, a larger value of the bombarding energy is needed to overcome the reaction barrier which is determined by the nucleus-nucleus potential and the dynamic barriers if they exist. However, the excitation energy of the compound nucleus increases with the bombarding energy. It decreases the survival probability relative to fission of a nucleus produced in a reaction and therefore puts a restriction on the upper value of the bombarding energy. To determine the optimal value of $E_{\rm c.m.}$ it is necessary to analyse a fusion process leading to compound nucleus formation. It consists of *i*) capture of nuclei, which can be dynamically deformed at the approach stage; *ii*) evolution of a dinuclear system formed to fusion in a competition with quasifission; *iii*) deexcitation (or fission) of a compound nucleus. The aim of the present paper is to calculate a capture probability and then to analyse a possible consequences of the results obtained for the fusion probability. To do it, we require in dynamic model to describe the initial stage of a heavy ion collision. Such a model has been developed in our earlier papers [1,2]. The capture probability is determined by the dynamic aspects of the reaction mechanism and by the depth of the pocket in the nucleus-nucleus interaction potential. As the examples we consider below the following reactions: ^{48}Ca + ^{244}Pu and ^{74,76}Ge + ^{208}Pb.

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2 Basic formalism

The cross section of production of the evaporation residues ($\sigma_{\rm ER}$)

$$\sigma_{\rm ER}(E) = \sum_{l=0}^{\infty} \sigma_l^{\rm fus}(E,l) W_{\rm sur}(E,l) \qquad (1)$$

is determined by the partial fusion cross section ($\sigma_l^{\rm fus}(E)$), as well as by the probability $W_{\rm sur}(E,l)$ that the compound nucleus survives during a deexitation cascade. In Eq. (1)

$$\sigma_l^{\rm fus}(E) = \sigma_l^{\rm capture}(E) P_{CN}(E,l), \qquad (2)$$

$$\sigma_l^{\rm capture}(E) = \frac{\lambda^2}{4\pi} \mathcal{P}_l^{\rm capture}(E), \qquad (3)$$

where λ is a wavelength, $P_{\rm CN}(E,l)$ is a factor taking into account a decrease of the fusion probability due to dinuclear system break up before fusion, $\mathcal{P}_l^{\rm capture}(E)$ is the capture probability which depends on the collision dynamics and determines the amount of partial waves leading to capture.

The main aim of the present paper is to calculate capture probability $\mathcal{P}_l^{\rm capture}(E)$. To do it we will use a dynamic approach developed in [1,2]. In this approach, the system of equations is derived to describe the relative motion of colliding nuclei and an evolution of their intrinsic states during the heavy ion collisions. This system of equa-

tions split into two subsystems. The first set of equations describes a relative motion. These equations are

$$\mu(\mathbf{R}(t))\ \ddot{\mathbf{R}}_k + \sum_j \gamma_{kj}[\mathbf{R}(t)]\ \dot{\mathbf{R}}_j\ (t) = -\frac{\partial V[\mathbf{R}(t)]}{\partial \mathbf{R}_k} \qquad (4)$$

where $\mathbf{R}(t)$ is a relative motion coordinate and $\dot{\mathbf{R}}\ (t)$ is a corresponding velocity. The second set of equations describes an evolution of the occupation numbers of the single particle states. These equations will be shown below.

In the Eq. (4), $V(\mathbf{R})$ is a nucleus-nucleus potential. This is the first important ingredient of the Eq. (4). This potential consists in two terms

$$V(\mathbf{R}) = V_0(\mathbf{R}) + \delta V(\mathbf{R}), \qquad (5)$$

where $V_0(\mathbf{R})$ is a static potential calculated in a frozen density approximation and $\delta V(\mathbf{R})$ is a dynamic contribution to a nucleus-nucleus potential, which is due to a rearrangement of the densities of the interacting nuclei during a reaction. The nuclear part of the static nucleus-nucleus potential $V_0(\mathbf{R})$ is calculated using a double - folding procedure with the effective nucleon-nucleon forces suggested by Migdal [3] and the densities of the interacting nuclei taken in the Woods-Saxon form

$$\rho_i^{(0)}(\mathbf{r}, \mathbf{R}_i(t), \theta_i, \beta_2^{(i)}) = \left[1 + \exp\left(\frac{|\mathbf{r} - \mathbf{R}_i(t)| - R_{0i}[1 + \beta_2^{(i)} Y_{20}(\theta_i)]}{a}\right)\right]^{-1}. \qquad (6)$$

In Eq. (6) $i = 1, 2$ is an index distinguishing the interacting nuclei; $\mathbf{R}_i$ are the center of mass coordinates and $\mathbf{R}_{0i}$ are the half density radii of the interacting nuclei; $\beta_2^{(i)}$ are the quadrupole deformation parameters determinated by the $B(E2)$ to the first excited 2^+ state [4], and θ_i are the axial symmetry axes orientations relative to $\mathbf{R}(t)$. An inclusion of a dependence on θ_i into the expressions for the nuclear densities $\rho_i^{(0)}$ give us a possibility to consider fusion at different mutual orientations of the interacting nuclei. The expression needed to calculate $\delta V(\mathbf{R})$ is given below. The effective nucleon-nucleon forces introduced by Migdal [3] take into account a different interaction strength inside and outside of the nucleus and the diffuseness of the nuclear surface by a linear density dependence of the constants.

The second important ingredient of the Eq. (4) is the friction tensor γ_{kj}. The expression for the friction tensor have been derived applying a linear response theory to the description of heavy ion collisions. We have assumed that the Hamiltonian describing an intrinsic motion of the nucleons in the interacting nuclei and its coupling to the relative motion can be taken as a sum of the Hamiltonian of the noninteracting nucleons moving in the time dependent potential, and a residual interaction term. The time dependent single particle potential is taken as the sum of the single particle potentials of the interacting nuclei. Thus,

$$\hat{H}\left(\mathbf{R}(t)\right) = \sum_{i=1}^{A}\left(\frac{-\hbar^2}{2m}\Delta_i + \hat{V}_P[\mathbf{r}_i - \mathbf{R}(t)] + \hat{V}_T(\mathbf{r}_i)\right) + h_{\text{residual}}, \qquad (7)$$

where $A = A_P + A_T$ is the total number of nucleons in the system. In the second quantization representation

$$\hat{\mathcal{H}}\left(\mathbf{R}(t), \xi\right) = \sum_P \varepsilon_P\ \mathrm{a}_P^\dagger\, \mathrm{a}_P + \sum_T \varepsilon_T\ \mathrm{a}_T^\dagger\, \mathrm{a}_T + \sum_{i,i'} V_{ii'}\left(\mathbf{R}(t)\right)\ \mathrm{a}_i^\dagger\, \mathrm{a}_{i'} + h_{\text{residual}}, \qquad (8)$$

where

$$\sum_{i,i'} V_{ii'}\left(\mathbf{R}(t)\right)\ \mathrm{a}_i^\dagger\, \mathrm{a}_{i'} = \sum_{P,P'} \Lambda_{PP'}^{(T)}\left(\mathbf{R}(t)\right)\ \mathrm{a}_P^\dagger\, \mathrm{a}_{P'} + \sum_{T,T'} \Lambda_{TT'}^{(P)}\left(\mathbf{R}(t)\right)\ \mathrm{a}_T^\dagger\, \mathrm{a}_{T'} + \sum_{T,P} \mathrm{g}_{PT}\left(\mathbf{R}(t)\right)\ (\mathrm{a}_P^\dagger \mathrm{a}_T + \mathrm{a}_T^\dagger\, \mathrm{a}_P)\,. \qquad (9)$$

Here $P \equiv (n_P, j_P, l_P, m_P)$ and $T \equiv (n_T, j_T, l_T, m_T)$ are the sets of quantum numbers characterizing the single particle states in an isolated projectile and target nuclei, respectively. The other quantities are

$$\Lambda_{PP'}^{(T)}\left(\mathbf{R}(t)\right) = \langle P|V_T(\mathbf{r})|P'\rangle, \qquad (10)$$

$$\Lambda_{TT'}^{(P)}\left(\mathbf{R}(t)\right) = \langle T|V_P[\mathbf{r} - \mathbf{R}(t)]|T'\rangle, \qquad (11)$$

$$g_{PT}\left(\mathbf{R}(t)\right) = \frac{1}{2}\langle P|V_P[\mathbf{r} - \mathbf{R}(t)] + V_T(\mathbf{r})|T\rangle. \qquad (12)$$

In Eq.(8) $\epsilon_{P(T)}$ are single particle energies of the nonperturbed states in the projectile (target). The diagonal matrix elements $\Lambda_{PP}^{(T)}$ ($\Lambda_{TT}^{(P)}$) describe the corrections to $\epsilon_{P(T)}$. The nondiagonal matrix elements $\Lambda_{PP'}^{(T)}$ ($\Lambda_{TT'}^{(P)}$) generates the particle-hole excitations in the projectilelike (targetlike) nucleus. The matrix elements g_{PT} are responsible for nucleon exchange between reaction partners.

Basing on the Hamiltonian (8) and applying the formalism of the linear response theory we get the following expression for the friction tensor

$$\gamma_{kj}\left[\mathbf{R}(t)\right] = \sum_{i,i'} \frac{\partial V_{ii'}\left[\mathbf{R}(t)\right]}{\partial R_k} \frac{\partial V_{ii'}\left[\mathbf{R}(t)\right]}{\partial R_j}\, B_{ii'}^{(1)}(t), \qquad (13)$$

where

$$B_{ik}^{(n)}(t) = \frac{2}{\hbar}\int_0^t dt'(t-t')^n \exp\left(\frac{t'-t}{\tau_{ik}}\right) \times \sin\left[\omega_{ik}\left(\mathbf{R}(t')\right)(t-t')\right]\left[\tilde{n}_k(t') - \tilde{n}_i(t')\right], \qquad (14)$$

$$\hbar\omega_{ik} = \epsilon_i + \Lambda_{ii} - \epsilon_k - \Lambda_{kk}. \qquad (15)$$

Here $\tau_{ij} = \tau_i\tau_k/(\tau_i + \tau_k)$; τ_i is the parameter describing the damping of the single-particle motion. The expression

for τ_i is derived in the theory of quantum liquids [3,5] (see Appendix A). The physical meaning of τ is explained below. This parameter approximately describes the effect of the residual interaction in $\mathcal{H}$.

The dynamical contribution $\delta V[\mathbf{R}(t)]$ to the nucleus–nucleus potential is calculated using the expression

$$\delta V\left[\mathbf{R}(t)\right] = \sum_{i,i'} \frac{\partial V_{ii'}\left[\mathbf{R}(t)\right]}{\partial R} \frac{\partial V_{ii'}\left[\mathbf{R}(t)\right]}{\partial R} B^{(0)}_{ii'}(t), \qquad (16)$$

where $B^{(0)}_{ii'}(t)$ is given by Eq. (13).

The third important ingredient of the Eq. (4) is the reduced mass $\mu[\mathbf{R}(t)]$ which is calculated using the expression

$$\mu(\mathbf{R}) = mA_TA_P/(A_T + A_P) + \delta\mu(\mathbf{R}), \qquad (17)$$

where $\delta\mu(\mathbf{R})$ is the dynamic contribution to the reduced mass. This dynamic correction is calculated using the expression of the same type as found in the linear response theory

$$\delta\mu\left[\mathbf{R}(t)\right] = \sum_{i,i'} \frac{\partial V_{ii'}\left[\mathbf{R}(t)\right]}{\partial R} \frac{\partial V_{ii'}\left[\mathbf{R}(t)\right]}{\partial R} B^{(2)}_{ii'}(t), \qquad (18)$$

where $B^{(2)}_{ii'}(t)$ is given by Eq. (13).

Since the Eq. (4) is applied only to description of the initial stage of an interaction of colliding nuclei (capture probability) we suppose that this approximation is quite satisfactory.

To calculate all these quantities it is necessary to know the occupation numbers of the single particle states. Since the excitation energy of the interacting nuclei changes significantly during the course of the collision, it is necessary to take into account the time dependence of the occupation numbers. It has been done by a numerical solution of the corresponding equations, which has been derived in [2,6] starting from the von Neumann type equation for a density matrix and doing some approximations. Since an explicit allowance for the residual interaction is very complicated it is customary to take into account a two–particle collision term in the linearized form (τ approximation) [5–7]. Then the equation for density matrix takes the form

$$i\hbar\frac{\partial\hat{\tilde{n}}(t)}{\partial t} = [\hat{\mathcal{H}}(\mathbf{R}(t)), \hat{\tilde{n}}(t)] - \frac{i\hbar}{\tau}[\hat{\tilde{n}}(t) - \hat{\tilde{n}}^{\rm eq}(\mathbf{R}(t))]\,, \qquad (19)$$

where $\tilde{n}^{\rm eq}(\mathbf{R}(t))$ is the local quasi-equilibrium distribution function of nucleons over the single particle state, *i.e.* a Fermi distribution with the temperature $\Theta(t)$ corresponding to the excitation energy at the internuclear distance $\mathbf{R}(t)$. In derivation of the final equation for the diagonal matrix elements of $\tilde{n}(t)$, which are the occupation numbers of the single particle states $n_i(t)$, it was assumed also that the phases of the nondiagonal matrix elements of $\tilde{n}(t)$ are chaotic.

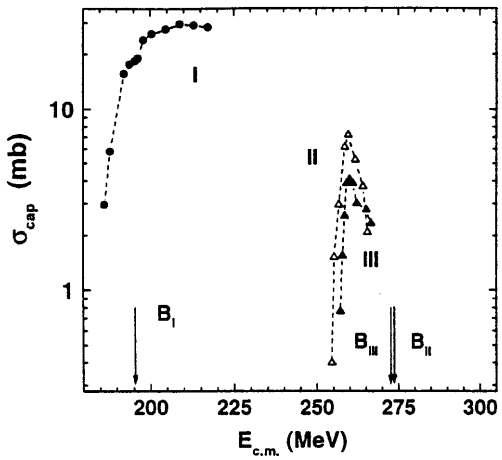

Fig. 1. The calculated capture cross section as a function of the beam energy for the ^{48}Ca + ^{244}Pu (I) (full circles), ^{74}Ge + ^{208}Pb (II) (full triangles), and ^{76}Ge + ^{208}Pb (III) (open triangles) reactions; B_i is the Bass barrier for the reaction (i), i=I, II, and III

3 Results and discussion

3.1 Capture cross section

We consider below the following reactions which are discussed now as possible ways to search for superheavy element with $Z = 114$. They are ^{48}Ca + ^{244}Pu (I) and ^{74,76}Ge + ^{208}Pb (II,III).

Basing on the dynamic model developed in [1] (which is described concisely in the preceding section) we have calculated the capture cross section $\sigma_l^{\rm capture}(E)$ for the reactions under consideration. The results are shown in Fig. 1. It is seen that, for these reactions there is an energy window for the values of the bombarding energy at which a capture cross section is large enough to have a physical interest. The lower limit for the bombarding energy ($E_{\rm min}$) is defined by a total nucleus-nucleus interaction potential $V(R) = V_0(R) + \delta V(R)$. Note that $E_{\rm min}$ is somewhat larger than the value of the entrance Coulomb barrier, because of the kinetic energy loss due to friction. So, $E_{\rm min}$ is determined by a dynamic calculation. The upper limit ($E_{\rm max}$) comes from an incomplete dissipation of the relative kinetic energy. Thus, the values of $E_{\rm min}$ and $E_{\rm max}$ are determined by the depth of the pocket in the potential $V(R)$ (Fig. 2) and by dissipative forces. If a bombarding energy is larger than $E_{\rm max}$ the dissipative forces could not provide a complete dissipation of the relative kinetic energy and dinuclear system decays into two fragments instead of being fused. As it is seen from Fig. 1, reaction with the lighter projectile (I) has a larger value of the capture cross section than other two reactions (II) and (III). The reason is that for ^{48}Ca + ^{244}Pu reaction the pocket of the nucleus–nucleus interaction potential is deeper and wider than for ^{74,76}Ge + ^{208}Pb (see Fig. 2).

The potentials presented in Fig. 2 are calculated taking into account a deformation of the interacting nuclei assum-

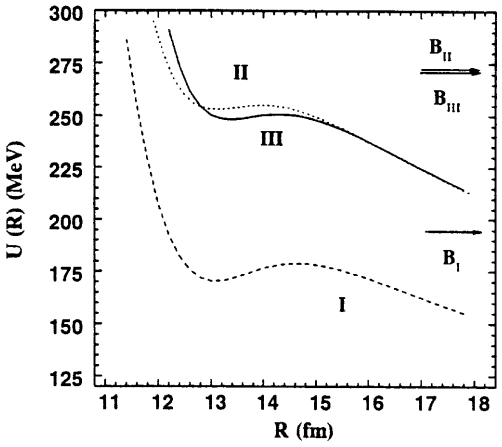

Fig. 2. The nucleus-nucleus interaction potential calculated for ^{76}Ge + ^{208}Pb (solid curve), ^{74}Ge + ^{208}Pb (dotted curve) and ^{48}Ca + ^{244}Pu (dashed curve) reactions; B_i is the Bass barrier for the reaction (i), i=I, II, and III

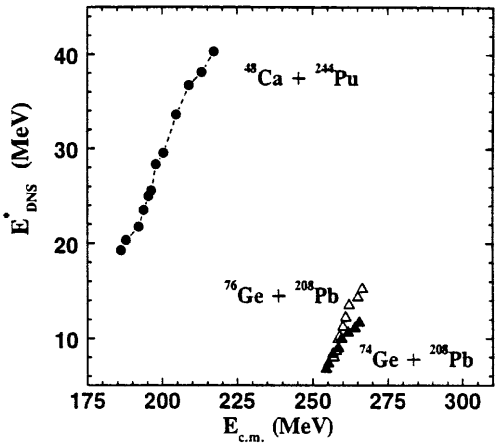

Fig. 3. The excitation energy of a dinuclear system formed after capture of nuclei in reactions: ^{48}Ca + ^{244}Pu (full circles), ^{74}Ge + ^{208}Pb (full triangles), and ^{76}Ge + ^{208}Pb (open triangles) as a function of the beam energy in the center of mass system

ing the tip-tip orientation of the colliding projectile and target nuclei. For other orientations of the colliding nuclei the potential is more flat and the depth of the pocket is smaller. Moreover, in these cases an entrance barrier and the minimum of the pocket of $V(R)$ have larger absolute energies than in the case of the tip-tip orientation. Therefore, an excitation energy of a compound nucleus will be larger than in the last case. An excess of the excitation energy will decrease the survival probability of the evaporation residues. Thus, in the fusion of massive nuclei their mutual orientation strongly influences not only the capture cross section but also the probability that the compound nucleus survives during deexcitation.

The existence of the window for the bombarding energy has a crucial influence on the fusion process. From one side a larger bombarding energy will be needed to overcome an intrinsic barrier ($B^*_{\rm fus}$) to form a compound nucleus. From other side an increase of the bombarding energy decreases the capture probability starting from some values of the bombarding energy because the friction force is not strong enough to provide a complete dissipation of the kinetic energy.

3.2 Excitation energy

To overcome an inner barrier for fusion a dinuclear system should have the corresponding excitation energy. However, the possible values of the excitation energy of a dinuclear system $E^*_{\rm DNS}$ which are defined by the amount of a dissipated energy are restricted by the framework of the energy window for bombarding energies leading to capture. The possible values of the excitation energy can be estimated using the expression

$$E^*_{\rm DNS} = E_{\rm c.m.} - V(R_m), \qquad (20)$$

where $V(R_m)$ is the value of the nucleus-nucleus interaction potential at the minimum. The results are shown in Fig. 3. For ^{48}Ca + ^{244}Pu reaction the excitation energy of a dinuclear system can take the values from 19 MeV up to 41 MeV.

In the case of ^{74,76}Ge + ^{208}Pb reactions, the excitation energy $E^*_{\rm DNS}$ takes the values between 6 MeV and 16 MeV. These values of a dinuclear system excitation energy should be compared with the values of the inner barrier for fusion in the models of fusion process. An increase of the beam energy in order to obtain a larger excitation energy does not help because dinuclear system can not be formed. The corresponding value of the beam energy will exceed $E_{\rm max}$.

3.3 Remarks on fusion probability

To analyse a fusion process further we need in a dynamic model which describes an evolution of a dinuclear system to compound nucleus formation. Below we will use a model developed in [8] which is successful in an explanation of a data on fusion of massive nuclei [9]. According to this model a dinuclear system evolves to the compound nucleus by increasing its mass asymmetry. Therefore, the carefully calculated driving potential plays the main role in a fusion dynamics. It was calculated as follows,

$$U(Z,A;l) = B_1(Z;A) + B_2(Z_P + Z_T - Z; A_P + A_T - A) + V(Z,A;R_m,l) - B_0, \qquad (21)$$

where B_1 and B_2 are the binding energies [10,11] of the nuclei in a dinuclear system, $V(R_m)$ is the value of the nucleus–nucleus interaction potential at the minimum, B_0 is the binding energy of the compound nucleus. For the given charge asymmetry the A/Z ratio was determined from the minimum value of $U(Z,A;l)$. So, a dinuclear system, which is formed after the capture at initial stage,

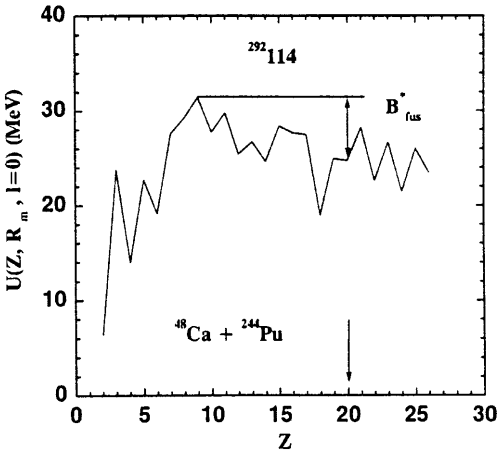

Fig. 4. The driving potential for the superheavy element $^{292}114$. The arrow indicates an initial charge asymmetry which corresponds to the ^{48}Ca + ^{244}Pu reaction

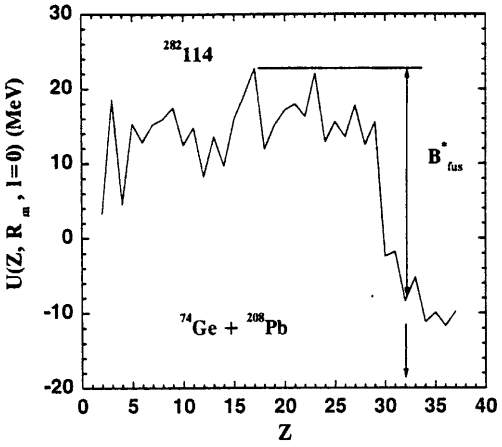

Fig. 5. The driving potential for the superheavy element $^{282}114$. The arrow indicates an initial charge asymmetry which corresponds to ^{74}Ge + ^{208}Pb reaction

should overcome the maximum of the driving potential to be fused. The calculated driving potentials for the reactions which we consider are presented in Figs. 4-6. As it is seen the values of the barriers which should be overcome to be fused ($B^*_{\rm fus}$) depend on the compound system and the reaction choice which determines the initial value of the mass asymmetry. These barriers are equal to 7 MeV for ^{48}Ca + ^{244}Pu (Fig. 4) and 30, 32 MeV for ^{74,76}Ge + ^{208}Pb (Figs. 5 and 6), respectively. Thus, in ^{48}Ca + ^{244}Pu reaction the excitation energy of the dinuclear system which takes the values between 19 MeV and 41 MeV is larger than the barrier $B^*_{\rm fus}$ of the driving potential. In the case of ^{74,76}Ge + ^{208}Pb reaction the excitation energy which takes the values between 6 MeV and 16 MeV is lower than the value of $B^*_{\rm fus}$ for these reactions. Thus, according to our calculations of a capture cross section and the model of fusion suggested in [8], the compound nucleus can not be formed with a measurable cross section in the ^{74,76}Ge + ^{208}Pb reactions. However, it is not excluded that a dinuclear system can prefer the trajectory in the $R-Z$ plane for fusion different from that suggested in [8] or other mechanism of the compound nucleus formation like cluster transfer [12] might play an important role.

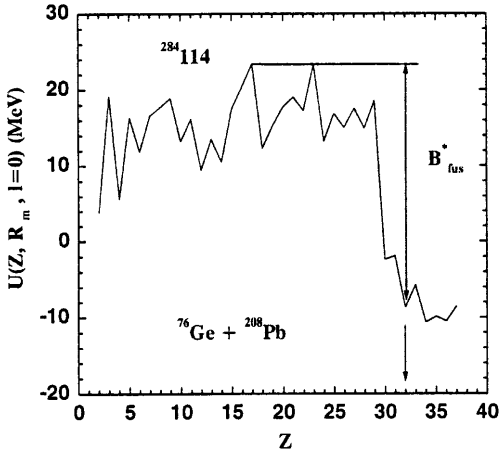

Fig. 6. The driving potential for the superheavy element $^{284}114$. The arrow indicates an initial charge asymmetry which corresponds to ^{76}Ge + ^{208}Pb reaction

The other question concerns the probability that the excited compound nucleus formed in a fusion process survives during deexcitation. An increase of an excitation energy decreases the influence of the shell effects on stability of a compound nucleus and decrease the fusion probability. However, this question is not analysed in the present paper.

4 Conclusion

We have analysed the partial fusion cross sections for the reactions with massive nuclei leading to compound nucleus with $Z = 114$: ^{48}Ca + ^{244}Pu and ^{74,76}Ge + ^{208}Pb. The main attention is paid to the calculations of the capture probability, which is a characteristic feature of an initial stage of the collision. It is shown that for the considered reactions, there is an energy window for the bombarding energy at which the capture cross section is large enough to have a physical interest. This result puts a strong limitations on the choice of the bombarding energy for a given reaction. However, from other side, the excitation energy should be large enough to overcome an intrinsic barrier for the fusion [8]. Thus, both restrictions can be used to obtain an optimal choice of the projectile-target combination and of the bombarding energy.

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Appendix A

The value of τ_i is calculated using the results of the theory of quantum liquids [3,5]

$$\frac{1}{\tau_i^{(\alpha)}} = \frac{\sqrt{2}\pi}{32\hbar\varepsilon_{F_K}^{(\alpha)}}\left[(f_K-g)^2+\frac{1}{2}(f_K+g)^2\right] \times\left[\left(\pi\Theta_K\right)^2+\left(\tilde{\varepsilon}_i-\lambda_K^{(\alpha)}\right)^2\right] \times\left[1+\exp\Big(\frac{\lambda_K^{(\alpha)}-\tilde{\varepsilon}_i}{\Theta_K}\Big)\right]^{-1}, \quad \text{(A1)}$$

where

$$\Theta_K(t)=3.46\sqrt{\frac{E_K^*(t)}{<A_K(t)>}}$$

is the effective temperature determined by the amount of intrinsic excitation energy $E_K^* = E_K^{*(Z)} + E_K^{*(N)}$; $< A_K(t) >=< Z_K(t) > + < N_K(t) >$, $\lambda_K^{(\alpha)}(t)$, and $E_K^{*(\alpha)}(t)$ are the mass number, chemical potential, and intrinsic excitation energies for the proton ($\alpha = Z$) and neutron ($\alpha = N$) subsystems of the nucleus $K(K = P,T)$, respectively, (for details, see [6]). Furthermore, the finite size of nuclei and the available difference between the numbers of neutrons and protons need to use the following expressions for the Fermi energies [3]:

$$\varepsilon_{F_K}^{(Z)} = \varepsilon_F\left[1-\frac{2}{3}\big(1+2f'\big)\frac{<N_K>-<Z_K>}{<A_K>}\right], \quad \text{(A2)}$$

$$\varepsilon_{F_K}^{(N)} = \varepsilon_F\left[1+\frac{2}{3}\big(1+2f'\big)\frac{<N_K>-<Z_K>}{<A_K>}\right], \quad \text{(A3)}$$

where ϵ_F=37 MeV,

$$f_K = f_{in} - \frac{2}{<A_K>^{1/3}}(f_{in}-f_{ex}),$$

$$f'_K = f'_{in} - \frac{2}{<A_K>^{1/3}}(f'_{in}-f'_{ex})$$

and f_{in}=0.09, f'_{in}=0.42, f_{ex}=-2.59, f'_{ex}=0.54, g=0.7 are the constants of the effective nucleon-nucleon interaction.

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